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Tests of CPT invariance for neutral flavored meson–antimeson mixing

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Abstract. We focus on two aspects of CPT invariance in neutral meson–antimeson ($M^0\bar{M}^0$) mixing: (1) tests of CPT invariance, using only the property of "lack of vacuum regeneration", which occurs as a part of the well-known Lee–Oehme–Yang (LOY) theory; (2) methods for extracting the CPT-violating mixing parameter θ through explicit calculations by fully using the LOY-type theory. In the latter context, we demonstrate the importance of the C-even $|M^0\bar{M}^0\rangle$ state. In particular, by measuring the time dependence of opposite-sign dilepton events arising from decays of the C-even and C-odd $|M^0\bar{M}^0\rangle$ states, θ may be disentangled from the parameters λ_+ and $\bar{\lambda}_-$ characterizing violations of the $\Delta F = \Delta Q$ rule. Furthermore, these two parameters may also be determined. The same is true if one uses like-sign dilepton events arising from only the C-even $|M^0\bar{M}^0\rangle$ state.

1 Introduction

The usual phenomenology of the complex formed by the neutral flavored meson M^0 ($M^0 = K^0$, D^0, B_d^0, B_s^0) and its antiparticle $\bar{M}^0$ is based on the Weisskopf–Wigner approximation (WWA) which is incorporated into theories of the Lee–Oehme–Yang (LOY) type [1–3]. This complex is investigated extensively for valuable studies like those of the discrete symmetries CP, T and CPT, and of physics beyond the standard model (for a review, see [4]). So far, the only known CP and T non-invariances have arisen in measurements on the $M^0\bar{M}^0$ complex, while CPT conservation is at present consistent with all existing data [5]. Therefore, testing CPT invariance at the phenomenological level is an important issue (see, e.g., [6]). The purpose of this paper is to consider some tests of CPT invariance in the mixing of M^0 and $\bar{M}^0$, at two levels [7] of the WWA. We will consider the following situations:

(1) transitions of single M^0 and $\bar{M}^0$ mesons into M^0 or $\bar{M}^0$; this would require flavor tagging of the initial *and* final states;
(2) transitions of single M^0 and $\bar{M}^0$ mesons into decay channels (e.g., $\pi\pi$, $\pi\ell\nu, \ldots$); here, only the initial states have to be tagged for flavor;
(3) transitions of the C-even and C-odd correlated $|M^0\bar{M}^0\rangle$ states into two flavored mesons ($M^0, \bar{M}^0$); this would require flavor tagging of the final states;
(4) transitions of these correlated states into decay channels, without need for flavor tagging.

We will demonstrate the importance of the C-even state [8] – particularly for disentangling CPT violation from violation of the $\Delta F = \Delta Q$ rule (F means flavor and may be S or C or B) in semileptonic decays; both these violations could arise from physics beyond the standard model. One may note that it is important to allow new physics through violations of the $\Delta F = \Delta Q$ rule if one is looking for new physics through CPT violation, especially because of the similarity [9,10] of the effects of these two types of violations.

Let us briefly mention some tagging methods. For neutral kaons, the CPLEAR [11] reactions $\bar{p}p \to K^+\pi^-\bar{K}^0$ $K^-\pi^+K^0$ allow flavor tagging of the initial kaon by utilizing the identity of the accompanying charged kaons and pions. This method is based only on strangeness conservation in strong interactions. Similarly, the reactions [12] $K^0p \to K^+n$, $\bar{K}^0p \to \pi^+\Lambda$ (see also [13]) may be used for final-state tagging, also for decays of the correlated $|K^0\bar{K}^0\rangle$ states. For heavier flavors, one may also do final-state tagging by using the flavor-conserving strong interactions – e.g., the "jet charge method" (see, e.g., [14]) corresponding to the relevant flavored quark. This procedure, used for B mesons, is a purely empirical procedure of the "calibrated" type, wherein, briefly speaking, one estimates the sign of the charge of the parent flavored quark by performing suitable weighted averages over charges of the particle tracks in the jet produced by the flavored quark; to make the analysis more reliable, the jets from the parent quark and the parent antiquark are simultaneously considered. Apart from its empirical nature, the procedure is general.

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2 Outline and formalism

The WWA is characterized by the introduction of two independently propagating states $|M_{H,L}\rangle$ which are linear combinations of the flavor states:

$$\begin{aligned} |M_H\rangle &= p_H|M^0\rangle + q_H|\bar{M}^0\rangle,\ |p_H|^2+|q_H|^2=1, \\ |M_L\rangle &= p_L|M^0\rangle - q_L|\bar{M}^0\rangle,\ \ |p_L|^2+|q_L|^2=1, \end{aligned} \tag{1}$$

where $p_{H,L}$ and $q_{H,L}$ are complex constants. Thus the time evolution is described by

$$|M_{H,L}\rangle \stackrel{t}{\to} \Theta_{H,L}(t)|M_{H,L}\rangle \quad \text{with} \quad \Theta_{H,L}(0)=1, \tag{2}$$

where t is the proper time and the $\Theta_{H,L}$ are the propagation functions. By the same token, a crucial property of the WWA is the *lack of vacuum regeneration* (called LVR below), i.e, the absence of transitions $|M_{H,L}\rangle \to |M_{L,H}\rangle$ in the time evolution. Let us define the general probability amplitudes for the transitions $|M^0\rangle \to |M^0\rangle$, $|M^0\rangle \to |\bar{M}^0\rangle$, $|\bar{M}^0\rangle \to |M^0\rangle$ and $|\bar{M}^0\rangle \to |\bar{M}^0\rangle$, respectively, as $a(t)$, $b(t)$, $\bar{b}(t)$ and $\bar{a}(t)$. Then LVR gives [7]

$$\bar{b}(t) = \alpha b(t), \tag{3}$$

$$\bar{a}(t) - a(t) = \beta b(t), \tag{4}$$

where α and β are complex constants determined in terms of $p_{H,L}$ and $q_{H,L}$ by

$$\alpha = \frac{p_H p_L}{q_H q_L} \quad \text{and} \quad \beta = \frac{p_L}{q_L} - \frac{p_H}{q_H}. \tag{5}$$

Equations (3) and (4) may qualitatively be visualized as follows. Using (i) (2), (ii) linearity of the transformation of (1) and its inverse, and (iii) the general constraints $\bar{a}(0)=a(0)=1$, $b(0)=\bar{b}(0)=0$, one must have $b(t)$, $\bar{b}(t)$ and $\bar{a}(t)-a(t)$ all proportional to $\Theta_H(t)-\Theta_L(t)$. While $|\alpha| \neq 1$ signifies T non-invariance, $\beta \neq 0$ signifies CPT non-invariance. Using (3) and (4), the transition rates can be expressed as functions of only two amplitudes, say a and b; therefore, in any theory using the LVR (e.g., the LOY theory) these equations are often useful in algebraic manipulations. The remaining part of the WWA can be expressed as [7]

$$a + \bar{a} = \Theta_H + \Theta_L, \tag{6}$$

$$b = q_H q_L (\Theta_H - \Theta_L)/D, \tag{7}$$

with $D = p_H q_L + p_L q_H$.

It is useful to define [4]

$$\theta = \frac{q_H/p_H - q_L/p_L}{q_H/p_H + q_L/p_L} \quad \text{and} \quad \frac{q}{p} = \sqrt{\frac{q_H q_L}{p_H p_L}}, \tag{8}$$

where both the real and imaginary parts of the phase-convention-independent parameter θ are in principle measurable and violate both CP and CPT; the quantity $|q/p|-1$ is a measure of CP and T violation in mixing. Then we can write

$$\alpha = \left(\frac{p}{q}\right)^2 \quad \text{and} \quad \beta = 2\frac{p}{q}\frac{\theta}{\sqrt{1-\theta^2}}. \tag{9}$$

With the parameters of (8), the description of the amplitudes a, b, $\bar{a}$, $\bar{b}$ in the full WWA is obtained as [4,15]

$$\begin{aligned} a(t) &= g_+(t) - \theta g_-(t), \\ b(t) &= \frac{q}{p}\sqrt{1-\theta^2}\, g_-(t), \\ \bar{a}(t) &= g_+(t) + \theta g_-(t), \\ \bar{b}(t) &= \frac{p}{q}\sqrt{1-\theta^2}\, g_-(t), \end{aligned} \tag{10}$$

where the functions $g_\pm$ are given by

$$g_\pm(t) = \frac{1}{2}\left(\Theta_H(t) \pm \Theta_L(t)\right). \tag{11}$$

So far, the functions $\Theta_{H,L}$ have not been specified. The exponential decay law of the WWA gives $\Theta_{H,L}$ as

$$\Theta_{H,L}(t) = \exp(-\mathrm{i}t\lambda_{H,L}) \quad \text{with} \quad \lambda_{H,L} = m_{H,L} - \frac{\mathrm{i}}{2}\Gamma_{H,L}, \tag{12}$$

where, as usual, $m_{H,L}$ are the real masses and $\Gamma_{H,L}$ the real decay widths of $|M_{H,L}\rangle$. In the following we will also need the definitions $\Delta m = m_H - m_L$ and $\Delta\Gamma = \Gamma_H - \Gamma_L$. We shall use the expression "full WWA" for (10), (11) and (12).

It is worth remarking that for unknown α and β, (3) and (4) provide tests of the LVR property. T (and CP) invariance gives, in general,

$$|b| = |\bar{b}|, \tag{13}$$

and CPT (and CP) invariance requires, in general,

$$a = \bar{a}. \tag{14}$$

Then, within the LVR, (13) means $|\alpha|=1$ and (14) means $\beta=0$. If CPT invariance, viz. (14) holds, it is not possible to test the proportionality of $(\bar{a}-a)$ to b, which is a characteristic of LVR. Thus, CPT invariance within the LVR means (14) and (3); the characteristic LVR form of (4) is then not relevant.

In the following sections, we focus on two subjects: (1) tests of CPT invariance within the LVR, (2) explicit calculations using the full WWA, with the aim of determining θ. In Sect. 3, we consider the one-time transitions described by the four amplitudes a, b, $\bar{a}$, $\bar{b}$. In Sect. 4 we summarize, for reference and comparison, decays of single M^0 and $\bar{M}^0$ mesons, which have been extensively investigated; see, e.g., [9]. In Sect. 5, the two-time transitions of the C-even and C-odd correlated $M^0\bar{M}^0$ states $|\psi_\pm\rangle$ to M^0M^0, $\bar{M}^0\bar{M}^0$, $M^0\bar{M}^0$ and $\bar{M}^0M^0$ final states are considered. Section 6 is devoted to the two-time decays of the correlated states $|\psi_\pm\rangle$ into physical channels f and g. Section 7 deals with explicit calculations by choosing f and g as semileptonic channels – both like-sign and opposite-sign dilepton events. Finally, Sect. 8 gives a summary.

3 Transitions of single mesons M^0 or $\bar{M}^0$ to M^0, $\bar{M}^0$

Let us first consider transitions of single M^0 or $\bar{M}^0$ mesons to M^0, $\bar{M}^0$, in analogy to the corresponding T invariance considerations of [11,16,17]. It has been argued [18] that in order to avoid further assumptions (arising from the use of weak-interaction decays as substitutes for flavor tags) in the interpretation [18–20] of the data [11], one should directly measure $|a|$, $|b|$, $|\bar{a}|$ and $|\bar{b}|$, and construct asymmetries out of these. In particular, the experimentally interesting asymmetries

$$K \equiv \frac{|\bar{b}|^2 - |b|^2}{|\bar{b}|^2 + |b|^2} \quad \text{and} \quad A \equiv \frac{|\bar{a}|^2 - |a|^2}{|\bar{a}|^2 + |a|^2} \tag{15}$$

test T invariance and CPT invariance, respectively: $K = 0$ and $A = 0$. While the LVR relation (3) involving the time-reversal parameter α gives a clear prediction for K, namely [16,17]

$$K = \frac{|\alpha|^2 - 1}{|\alpha|^2 + 1} = \text{constant}, \tag{16}$$

the corresponding LVR relation (4) involving the CPT parameter β does not give a simple and testable prediction for A, because β and b are not rephasing-invariant [4] and the t-dependent rephasing-invariant product βb is not easily accessible. However, the LVR relation (4) may be used to get the bound

$$-|\beta| \leq \frac{|\bar{a}| - |a|}{|b|} \leq |\beta|. \tag{17}$$

Unfortunately, this bound is not a clean equality test, in contrast to (16).

If we use the full WWA, the appropriate CPT observable A is obtained as

$$A = \frac{|\bar{a}|^2 - |a|^2}{|\bar{a}|^2 + |a|^2} = 2\mathrm{Re}\left[\theta \frac{g_-(t)}{g_+(t)}\right] \tag{18}$$

to first order in the CPT-violating parameter θ.

4 Decays of single M^0, $\bar{M}^0$ mesons

We investigate now the decays $|M^0(t)\rangle \to |f\rangle$ and $|\bar{M}^0(t)\rangle \to |f\rangle$, where $|M^0(0)\rangle = |M^0\rangle$ and $|\bar{M}^0(0)\rangle = |\bar{M}^0\rangle$. These have the decay rates

$$R(f,t) = |\langle f|T|M^0(t)\rangle|^2 = \left|a(t)A_f + b(t)\bar{A}_f\right|^2, \tag{19}$$

$$\bar{R}(f,t) = |\langle f|T|\bar{M}^0(t)\rangle|^2 = \left|\bar{a}(t)\bar{A}_f + \bar{b}(t)A_f\right|^2, \tag{20}$$

where we have used the definitions

$$\langle f|T|M^0\rangle = A_f, \quad \langle f|T|\bar{M}^0\rangle = \bar{A}_f. \tag{21}$$

These decays have been discussed previously in the light of CPT violation in mixing; see, e.g., [9]. We review them here for comparison with our alternative method in Sect. 7.

In order to exploit the rates (19) and (20) for the determination of θ, it is necessary to have information on the decay amplitudes A_f and $\bar{A}_f$. Let us focus on semileptonic decays with final states $X\ell^+\nu_\ell$ and $\bar{X}\ell^-\bar{\nu}_\ell$, where X $(\bar{X})$ is a specific hadronic state. Allowing for transitions which violate the $\Delta F = \Delta Q$ rule, we introduce the rephasing-invariant quantities [4]

$$\lambda_+ = \frac{q}{p}\frac{\bar{A}_+}{A_+} \quad \text{and} \quad \bar{\lambda}_- = \frac{p}{q}\frac{A_-}{\bar{A}_-}, \tag{22}$$

where $A_+ \equiv A_{\ell^+}$, $\bar{A}_- \equiv \bar{A}_{\ell^-}$, and so on. For instance, the CPLEAR Collaboration in [21] considers semileptonic decays of tagged K^0 and $\bar{K}^0$ with $X = \pi^-$. Using the convenient notation $R_+(t)$ for having M^0 at $t = 0$ decaying semileptonically into ℓ^+, etc., one obtains [9]

$$R_+(t) = |A_+|^2 \left|g_+(t) + g_-(t)\left(\lambda_+\sqrt{1-\theta^2} - \theta\right)\right|^2, \tag{23}$$

$$\bar{R}_-(t) = |\bar{A}_-|^2 \left|g_+(t) + g_-(t)\left(\bar{\lambda}_-\sqrt{1-\theta^2} + \theta\right)\right|^2, \tag{24}$$

$$R_-(t) = |\bar{A}_-|^2 \left|\frac{q}{p}\right|^2 \times \left|g_+(t)\bar{\lambda}_- + g_-(t)\left(\sqrt{1-\theta^2} - \theta\bar{\lambda}_-\right)\right|^2, \tag{25}$$

$$\bar{R}_+(t) = |A_+|^2 \left|\frac{p}{q}\right|^2 \times \left|g_+(t)\lambda_+ + g_-(t)\left(\sqrt{1-\theta^2} + \theta\lambda_+\right)\right|^2. \tag{26}$$

These four rates allow one to disentangle CPT violation in mixing from violations of the $\Delta F = \Delta Q$ rule [9].

In order to get a feeling for the experimental results of [21], it is useful to compare the rates (23), (24), (25), (26) with the corresponding ones in (9a)–(9d) of [21], which were expressed there by using a particular rephasing non-invariant parameterization. One finds the correspondences $\lambda_+ \leftrightarrow -x$, $\bar{\lambda}_- \leftrightarrow -\bar{x}^*$, $\theta \leftrightarrow 2\delta$, $(1 - |q/p|)/2 \leftrightarrow \mathrm{Re}\varepsilon$, $(1 - |\bar{A}_-/A_+|^2)/4 \leftrightarrow \mathrm{Re}y$; all these supposedly small parameters were retained up to only the first order. Note that the short-lived and long-lived kaons correspond, respectively, to our states $|M_L\rangle$ and $|M_H\rangle$. Among the three asymmetries constructed out of the four K^0_{e3} decay rates in [21], the one relevant for the determination of θ is their $A_\delta(t)$, which involves also the complex parameter $\bar{\lambda}_- - \lambda_+$. The result of [21] is

$$\begin{aligned} \mathrm{Re}\theta &= (6.0 \pm 6.6 \pm 1.2) \times 10^{-4}, \\ \mathrm{Im}\theta &= (-3.0 \pm 4.6 \pm 0.6) \times 10^{-2}, \\ \mathrm{Re}(\bar{\lambda}_- - \lambda_+) &= (0.4 \pm 2.6 \pm 0.6) \times 10^{-2}, \\ \mathrm{Im}(\bar{\lambda}_- - \lambda_+) &= (2.4 \pm 4.4 \pm 0.6) \times 10^{-2}, \end{aligned} \tag{27}$$

where the first error is statistical and the second is systematic. Though the experimental results (27) are consistent with $\theta = 0$ and $\bar{\lambda}_- - \lambda_+ = 0$, the large errors in these results could be concealing sizable violations of CPT invariance and of the $\Delta S = \Delta Q$ rule. For experimental reasons, the full information contained in the four rates $\overset{(-)}{R}_\pm(t)$ was not accessible, and it turns out that the two complex parameters $\bar{\lambda}_-$ and λ_+ were not fully separated [21]. Though the best $(K^0, \bar{K}^0)$ data presently available allow one to determine θ (with sizable errors), they are unable to determine separately the $\Delta S = \Delta Q$ rule-violating parameters; therefore, heavy $(M^0, \bar{M}^0)$ systems, wherein the clean CPLEAR method of flavor tagging is not applicable, are likely to pose more severe problems. Consequently, it is interesting to have – for the purpose of obtaining the above three complex parameters separately – an alternative procedure which does not require flavor tagging. We shall see in Sect. 7 that semileptonic decays of C-even correlated states, in addition to those of the C-odd ones, may provide such an alternative.

5 States of two mesons $(M^0, \bar{M}^0)$ arising from correlated states $|M^0\bar{M}^0\rangle$

Let us now consider the entangled states

$$|\psi_\epsilon\rangle = \frac{1}{\sqrt{2}}\Big[|M^0(\mathbf{k})\rangle \otimes |\bar{M}^0(-\mathbf{k})\rangle + \epsilon|\bar{M}^0(\mathbf{k})\rangle \otimes |M^0(-\mathbf{k})\rangle\Big], \tag{28}$$

where $\epsilon = \pm 1$ refers to the C-even and C-odd state, respectively. First we discuss probabilities for finding $|M^0(\mathbf{k})\rangle$ (the momentum $\mathbf{k}$ pointing to the left-hand side) at time t_ℓ and $|M^0(-\mathbf{k})\rangle$ at time t_r (on the right-hand side), etc. (see, e.g., [22]):

$$P_\epsilon(M^0, t_\ell; M^0, t_r) = \frac{1}{2}\left|a_\ell \bar{b}_r + \epsilon \bar{b}_\ell a_r\right|^2, \tag{29}$$

$$P_\epsilon(\bar{M}^0, t_\ell; \bar{M}^0, t_r) = \frac{1}{2}\left|b_\ell \bar{a}_r + \epsilon \bar{a}_\ell b_r\right|^2, \tag{30}$$

$$P_\epsilon(M^0, t_\ell; \bar{M}^0, t_r) = \frac{1}{2}\left|a_\ell \bar{a}_r + \epsilon \bar{b}_\ell b_r\right|^2, \tag{31}$$

$$P_\epsilon(\bar{M}^0, t_\ell; M^0, t_r) = \frac{1}{2}\left|b_\ell \bar{b}_r + \epsilon \bar{a}_\ell a_r\right|^2, \tag{32}$$

where $\overset{(-)}{a}_\ell \equiv \overset{(-)}{a}(t_\ell)$ and $\overset{(-)}{a}_r \equiv \overset{(-)}{a}(t_r)$, etc. No assumption of any discrete symmetry or about the LOY theory and WWA is made.

One may define the asymmetries [22]

$$Q_{1\epsilon}(t_\ell, t_r) = \frac{P_\epsilon(M^0, t_\ell; M^0, t_r) - P_\epsilon(\bar{M}^0, t_\ell; \bar{M}^0, t_r)}{P_\epsilon(M^0, t_\ell; M^0, t_r) + P_\epsilon(\bar{M}^0, t_\ell; \bar{M}^0, t_r)}, \tag{33}$$

$$Q_{2\epsilon}(t_\ell, t_r) = \frac{P_\epsilon(M^0, t_\ell; \bar{M}^0, t_r) - P_\epsilon(\bar{M}^0, t_\ell; M^0, t_r)}{P_\epsilon(M^0, t_\ell; \bar{M}^0, t_r) + P_\epsilon(\bar{M}^0, t_\ell; M^0, t_r)}. \tag{34}$$

For $\epsilon = -1$, using LVR fully, i.e., (3) and (4), one gets

$$Q_{1-}(t_\ell, t_r) = \frac{|\alpha|^2 - 1}{|\alpha|^2 + 1} = \text{constant}, \tag{35}$$

which equals the one-time asymmetry K of (16), as previously noted [23,24]; neither CPT invariance nor T invariance has been assumed.

Using CPT invariance and the LVR property, the C-even case ($\epsilon = +1$) also gives the result (35). Using LVR and then the WWA relations (10) for calculating Q_{1+}, we obtain

$$a_\ell b_r + b_\ell a_r = \frac{q}{p}\sqrt{1-\theta^2}\left[g_-(t_\ell + t_r) - 2\theta g_-(t_\ell) g_-(t_r)\right], \tag{36}$$

$$a_\ell b_r + b_\ell a_r + 2\beta b_\ell b_r = \frac{q}{p}\sqrt{1-\theta^2}\left[g_-(t_\ell + t_r) + 2\theta g_-(t_\ell) g_-(t_r)\right]; \tag{37}$$

this gives, to first order in the symmetry-violating parameters,

$$Q_{1+} \simeq \frac{|\alpha|^2 - 1}{|\alpha|^2 + 1} - 4\mathrm{Re}\left[\frac{\theta g_-(t_\ell) g_-(t_r)}{g_-(t_\ell + t_r)}\right]. \tag{38}$$

The first term on the right-hand side is, of course, identical with Q_{1-}. Therefore, it should be possible to extract θ from the time dependence of (38). In order to get a feeling for the second term on the right-hand side of (38), we consider two limiting cases. For $t_\ell, t_r \gg |2/\Delta\Gamma|$, one gets

$$Q_{1+} - Q_{1-} \to 2\mathrm{Re}\theta\,\mathrm{sign}(\Delta\Gamma). \tag{39}$$

On the other hand, for small times

$$t_\ell, t_r \ll 1/\sqrt{(\Delta m)^2 + (\Delta\Gamma/2)^2},$$

one can show that

$$Q_{1+} - Q_{1-} \to 2\mathrm{Re}\left[\theta\left(\mathrm{i}\Delta m + \frac{1}{2}\Delta\Gamma\right)\right]\frac{t_\ell t_r}{t_\ell + t_r}. \tag{40}$$

Now we come to the asymmetry $Q_{2\epsilon}$. Under the exchange $t_\ell \leftrightarrow t_r$, the probabilities (31) and (32) get exchanged, due to invariance under a 180° rotation. Using (3), one can see that $Q_{2\epsilon}$ is non-zero only if CPT invariance does not hold, viz. $a \neq \bar{a}$. This provides a test of CPT invariance within LVR, for both $\epsilon = \pm 1$: in the probabilities (31) and (32), the part which is odd under $t_\ell \leftrightarrow t_r$ vanishes.

The LVR relations of (3) and (4) give

$$P_\epsilon(M^0, t_\ell; \bar{M}^0, t_r) - P_\epsilon(\bar{M}^0, t_\ell; M^0, t_r) = \mathrm{Re}\left\{\left(a_\ell a_r + \epsilon\alpha b_\ell b_r + \beta\frac{1}{2}(a_\ell b_r + b_\ell a_r)\right)^* \times (a_\ell b_r - b_\ell a_r)\beta\right\}. \tag{41}$$

This difference (and also $Q_{2\epsilon}$) is non-zero only for $\beta \neq 0$. Invoking the full WWA gives the asymmetries $Q_{2\epsilon}$, to first order in θ, as

$$Q_{2-} \simeq 2\mathrm{Re}\left[\frac{\theta G_2(t_\ell, t_r)}{G_1(t_\ell, t_r)}\right] = -2\mathrm{Im}\left[\frac{\theta\sin(t_-\Delta\lambda/2)}{\cos(t_-\Delta\lambda/2)}\right], \tag{42}$$

$$Q_{2+} \simeq 2\mathrm{Re}\left[\frac{\theta G_2(t_\ell, t_r)}{g_+(t_\ell + t_r)}\right] = -2\mathrm{Im}\left[\frac{\theta\sin(t_-\Delta\lambda/2)}{\cos(t_+\Delta\lambda/2)}\right]. \tag{43}$$

Here, we have defined the complex parameter $\Delta\lambda = \lambda_H - \lambda_L = \Delta m - \mathrm{i}\Delta\Gamma/2$ and the real parameters $t_\pm = t_\ell \pm t_r$. Furthermore, $G_1(t_\ell, t_r)$ and $G_2(t_\ell, t_r)$ are, respectively, even and odd under $t_\ell \leftrightarrow t_r$:

$$
\begin{aligned}
G_{\substack{1\\2}}(t_\ell, t_r) &= g_+(t_\ell) g_\pm(t_r) - g_-(t_\ell) g_\mp(t_r) \\
&= \frac{1}{2}\left[\mathrm{e}^{-\mathrm{i}(\lambda_L t_\ell + \lambda_H t_r)} \pm \mathrm{e}^{-\mathrm{i}(\lambda_H t_\ell + \lambda_L t_r)}\right]. \quad (44)
\end{aligned}
$$

Again, one can see that the real and imaginary parts of θ can be extracted from measurements of Q_{2-} or Q_{2+}.

6 Decays of the correlated states into physical channels

We now come to the decays of $|\psi_\epsilon\rangle$ into the physical channel f detected at t_ℓ and the physical channel g detected at t_r. Then the rate is (see, e.g., [7]), assuming the closed nature of the $[(|M^0\rangle, |\bar{M}^0\rangle) \leftrightarrow (|M_H\rangle, |M_L\rangle)]$ system,

$$
\begin{aligned}
&R_\epsilon(f, t_\ell; g, t_r) \quad (45)\\
&= \frac{1}{2}\bigg|(a_\ell \bar{b}_r + \epsilon \bar{b}_\ell a_r) A_f A_g + (b_\ell \bar{a}_r + \epsilon \bar{a}_\ell b_r) \bar{A}_f \bar{A}_g \\
&+ \left(a_\ell \bar{a}_r + b_\ell \bar{b}_r + \epsilon(\bar{a}_\ell a_r + \bar{b}_\ell b_r)\right) \frac{1}{2}(A_f \bar{A}_g + \bar{A}_f A_g) \\
&+ \left(a_\ell \bar{a}_r - b_\ell \bar{b}_r - \epsilon(\bar{a}_\ell a_r - \bar{b}_\ell b_r)\right) \frac{1}{2}(A_f \bar{A}_g - \bar{A}_f A_g)\bigg|^2,
\end{aligned}
$$

wherein the transition amplitudes of (21) are used.

As for $P_\epsilon(M^0, t_\ell; \bar{M}^0, t_r)$, we observe that, in R_+, the part which is odd under $t_\ell \leftrightarrow t_r$ vanishes if CPT invariance holds within LVR. Within the full WWA, this result has been noted earlier in an explicit calculation [8]; the present result is based on simpler and more general considerations. Taking into account both (3) and (4), the rate R_+ is given by

$$
\begin{aligned}
R_+(f, t_\ell; g, t_r) = \frac{1}{2}\bigg|&(a_\ell b_r + b_\ell a_r) \\
&\times \left(\alpha A_f A_g + \bar{A}_f \bar{A}_g + \beta \frac{1}{2}(A_f \bar{A}_g + \bar{A}_f A_g)\right) \\
&+ 2 b_\ell b_r \left(\beta \bar{A}_f \bar{A}_g + \alpha \frac{1}{2}(A_f \bar{A}_g + \bar{A}_f A_g)\right) \\
&+ a_\ell a_r (A_f \bar{A}_g + \bar{A}_f A_g) \\
&+ (a_\ell b_r - b_\ell a_r) \beta \frac{1}{2}(A_f \bar{A}_g - \bar{A}_f A_g)\bigg|^2. \quad (46)
\end{aligned}
$$

This formula shows that, for $\beta \neq 0$, R_+ contains a part odd under $t_\ell \leftrightarrow t_r$.

With the full WWA, the rates $R_\mp$ assume the well-known forms [9]

$$
\begin{aligned}
&R_-(f, t_\ell; g, t_r) \\
&= \frac{1}{2}\bigg|\left[G_1(t_\ell, t_r) + \theta G_2(t_\ell, t_r)\right] A_f \bar{A}_g \\
&- \left[G_1(t_\ell, t_r) - \theta G_2(t_\ell, t_r)\right] \bar{A}_f A_g \\
&+ \sqrt{1-\theta^2}\, G_2(t_\ell, t_r) \left(\frac{p}{q} A_f A_g - \frac{q}{p} \bar{A}_f \bar{A}_g\right)\bigg|^2 \quad (47)
\end{aligned}
$$

and [8]

$$
\begin{aligned}
&R_+(f, t_\ell; g, t_r) \\
&= \frac{1}{2}\bigg|\left[g_+(t_+) - 2\theta^2 g_-(t_\ell)\, g_-(t_r)\right] \left(A_f \bar{A}_g + \bar{A}_f A_g\right) \\
&+ \frac{p}{q}\sqrt{1-\theta^2}\left[g_-(t_+) - 2\theta g_-(t_\ell)\, g_-(t_r)\right] A_f A_g \\
&+ \frac{q}{p}\sqrt{1-\theta^2}\left[g_-(t_+) + 2\theta g_-(t_\ell)\, g_-(t_r)\right] \bar{A}_f \bar{A}_g \\
&+ \theta G_2(t_\ell, t_r) \left(A_f \bar{A}_g - \bar{A}_f A_g\right)\bigg|^2. \quad (48)
\end{aligned}
$$

7 Dilepton events from correlated decays

For explicit calculations, we first consider opposite-sign dilepton events [10,25], i.e., semileptonic decays with $f = X\ell^+\nu_\ell$ and $g = \bar{X}\ell^-\bar{\nu}_\ell$. For illustrating our point, we consider the same type of lepton on the two sides. The amplitudes A_+, etc. and the $\Delta F = \Delta Q$ rule-violating parameters λ_+ and $\bar{\lambda}_-$ are defined in Sect. 4. We assume that the quantities θ, λ_+ and $\bar{\lambda}_-$, which describe "unexpected" physics, are small; we retain contributions up to only the first order in these quantities.

First, we want to show that, by observing the time dependence of decays into opposite-sign dilepton events of both $|\psi_+\rangle$ and $|\psi_-\rangle$, it is possible to disentangle θ, λ_+, and $\bar{\lambda}_-$. This is easily seen by comparing [9]

$$
\begin{aligned}
&R_-(\ell^+, t_\ell; \ell^-, t_r) \quad (49)\\
&= \frac{1}{2}|A_+|^2 |\bar{A}_-|^2 \left|G_1(t_\ell, t_r) + (\theta - \lambda_+ + \bar{\lambda}_-) G_2(t_\ell, t_r)\right|^2,
\end{aligned}
$$

with [8]

$$
\begin{aligned}
R_+(\ell^+, t_\ell; \ell^-, t_r) &= \frac{1}{2}|A_+|^2 |\bar{A}_-|^2 \\
&\times \left|g_+(t_+) + (\lambda_+ + \bar{\lambda}_-) g_-(t_+) + \theta G_2(t_\ell, t_r)\right|^2. \quad (50)
\end{aligned}
$$

The part of the rate $R_-(\ell^+, t_\ell; \ell^-, t_r)$ which is odd under $t_\ell \leftrightarrow t_r$ determines the combination $\theta - \lambda_+ + \bar{\lambda}_-$, whereas in the case of $R_+(\ell^+, t_\ell; \ell^-, t_r)$ the odd and even parts depend on θ and $\lambda_+ + \bar{\lambda}_-$, respectively.

Considering like-sign dilepton events [9,25], "new physics" does not enter at first order for the C-odd state [9]:

$$
R_-(\ell^+, t_\ell; \ell^+, t_r) = \frac{1}{2}|A_+|^4 \left|\frac{p}{q}\right|^2 |G_2(t_\ell, t_r)|^2 \quad (51)
$$

and

$$
R_-(\ell^-, t_\ell; \ell^-, t_r) = \frac{1}{2}|\bar{A}_-|^4 \left|\frac{q}{p}\right|^2 |G_2(t_\ell, t_r)|^2. \quad (52)
$$

However, correlated decays of the C-even state into like-sign dilepton events do contain "new physics" at first order:

$$R_+(\ell^+, t_\ell; \ell^+, t_r) \\ = \frac{1}{2}|A_+|^4 \left|\frac{p}{q}\right|^2 |g_-(t_+) + 2\lambda_+ g_+(t_+) - 2\theta g_-(t_\ell) g_-(t_r)|^2 \tag{53}$$

and

$$R_+(\ell^-, t_\ell; \ell^-, t_r) \\ = \frac{1}{2}|\bar{A}_-|^4 \left|\frac{q}{p}\right|^2 \left|g_-(t_+) + 2\bar{\lambda}_- g_+(t_+) + 2\theta g_-(t_\ell) g_-(t_r)\right|^2 . \tag{54}$$

From these two rates, which are obviously symmetric under $t_\ell \leftrightarrow t_r$, the quantities θ, λ_+ and $\bar{\lambda}_-$ could be disentangled because the functions of t_ℓ and t_r with which they are associated are different.

A remark is now in order concerning the comparison of the formulas of this section with experiment. In general, the amplitudes $A_\pm$, $\bar{A}_\pm$ will depend on the detailed configuration of the final state $X\ell^+\nu_\ell$ or $\bar{X}\ell^-\bar{\nu}_\ell$, i.e., on the particle content of X ($\bar{X}$) and the momenta and polarizations of all particles in the final states. Let us denote the sum over various choices of X ($\bar{X}$) and various configurations of spins and momenta detected on the left-hand side by $\langle \ldots \rangle_\ell$ and the corresponding sum detected on the right-hand side by $\langle \ldots \rangle_r$. Consider, as an example, the rate $R_-(\ell^+, t_\ell; \ell^+, t_r)$. Taking into consideration the summation over the final configurations, we obtain (see also [21,26])

$$\begin{aligned} &\langle R_-(\ell^+, t_\ell; \ell^+, t_r)\rangle_{\ell,r} \\ &= \frac{1}{2}\Bigg\{ |G_2(t_\ell, t_r)|^2 \left|\frac{p}{q}\right|^2 \langle |A_+|^2\rangle_\ell \langle |A_+|^2\rangle_r \\ &+ 2\mathrm{Re}\Bigg[G_2(t_\ell, t_r)^* G_1(t_\ell, t_r) \left(\frac{p}{q}\right)^* \\ &\times \left(\langle |A_+|^2\rangle_\ell \langle A_+^* \bar{A}_+\rangle_r - \langle A_+^* \bar{A}_+\rangle_\ell \langle |A_+|^2\rangle_r \right) \Bigg]\Bigg\} \\ &= \frac{1}{2}\left|\frac{p}{q}\right|^2 \langle |A_+|^2\rangle_\ell \langle |A_+|^2\rangle_r \\ &\times \left| G_2(t_\ell, t_r) + G_1(t_\ell, t_r)(\lambda_+^r - \lambda_+^\ell)\right|^2 , \end{aligned} \tag{55}$$

where at most the first order in the small parameters θ and

$$\lambda_+^r = \frac{q}{p}\frac{\langle A_+^* \bar{A}_+\rangle_r}{\langle |A_+|^2\rangle_r}, \quad \lambda_+^\ell = \frac{q}{p}\frac{\langle A_+^* \bar{A}_+\rangle_\ell}{\langle |A_+|^2\rangle_\ell} \tag{56}$$

has been retained. We similarly obtain

$$\begin{aligned} &\langle R_-(\ell^-, t_\ell; \ell^-, t_r)\rangle_{\ell,r} \\ &= \frac{1}{2}\left|\frac{q}{p}\right|^2 \langle |\bar{A}_-|^2\rangle_\ell \langle |\bar{A}_-|^2\rangle_r \\ &\times \left| G_2(t_\ell, t_r) + G_1(t_\ell, t_r)(\bar{\lambda}_-^r - \bar{\lambda}_-^\ell)\right|^2 , \end{aligned} \tag{57}$$

where

$$\bar{\lambda}_-^r = \frac{p}{q}\frac{\langle \bar{A}_-^* A_-\rangle_r}{\langle |\bar{A}_-|^2\rangle_r}, \quad \bar{\lambda}_-^\ell = \frac{p}{q}\frac{\langle \bar{A}_-^* A_-\rangle_\ell}{\langle |\bar{A}_-|^2\rangle_\ell}. \tag{58}$$

The ratio of the rates in (55) and (57) has a constant value if the $\Delta F = \Delta Q$ rule holds, in which case the lepton charge *is* the flavor tag; if, in addition, CPT invariance in the amplitudes holds and if for a given side (viz. left or right), the states and configurations summed over in (55) and (57) are CPT-conjugates of each other, the constant value is just the time-reversal parameter $|p/q|^4$; see [23] for corresponding remarks if tagging of the final flavored mesons is not replaced by their semileptonic decays. On the other hand, one now sees that, in $\langle R_-(\ell^+, t_\ell; \ell^+, t_r)\rangle_{\ell,r}$ and $\langle R_-(\ell^-, t_\ell; \ell^-, t_r)\rangle_{\ell,r}$, violations of the $\Delta F = \Delta Q$ rule cancel if left- and right-hand sides are summed over identical states and configurations. Implicitly, we have made this assumption of identical left and right summations in all our results in (49)–(54), where the $\Delta F = \Delta Q$ rule-violating parameters λ_+ and $\bar{\lambda}_-$ should be perceived as the effective parameters of (56) and (58), respectively (of course, now we have $\lambda_+^r = \lambda_+^\ell$ and $\bar{\lambda}_-^r = \bar{\lambda}_-^\ell$). Equations (55) and (57) illustrate this point for (51) and (52), respectively, and show the importance of identical left and right summations.

8 Conclusions

In this paper we have discussed two items. Firstly, we have proposed tests of CPT invariance within only the *lack of vacuum regeneration* (LVR) property. This means testing $a = \bar{a}$ and $\bar{b} \propto b$ together (see (14) and (3)). The second item is the determination, by assuming the full WWA, of the parameter θ of (8), which is a measure of CPT violation in $M^0\bar{M}^0$ mixing. In the following, subscripts $\mp$ refer to the C-odd and C-even $|M^0\bar{M}^0\rangle$ states $|\psi_\mp\rangle$ of (28), respectively.

As for the first point, we have noted the following qualitative tests.

(i) The asymmetry $Q_{1+}(t_\ell, t_r)$ (see (33)) equals $Q_{1-}(t_\ell, t_r)$ of (35); these are asymmetries for transitions into M^0M^0 and $\bar{M}^0\bar{M}^0$ final states.
(ii) The asymmetries $Q_{2\mp}(t_\ell, t_r)$ for $M^0\bar{M}^0$ and $\bar{M}^0M^0$ final states vanish. Correspondingly, the ($t_\ell \leftrightarrow t_r$)-odd parts of the probabilities (31) and (32) vanish.
(iii) The ($t_\ell \leftrightarrow t_r$)-odd part of the decay rate $R_+(f, t_\ell; g, t_r)$ of (46) vanishes.

The second item of our paper, viz. methods for the determination of θ, involves explicit computations of observables within the full WWA. These observables include the cases where $\theta = 0$ would reproduce one of the abovementioned tests, i.e., Q_{1+} in (38), Q_{2-} in (42), Q_{2+} in (43) and R_+ in (48). In addition, we have the rate R_- in (47) and, for one-time single meson transitions, the asymmetry A in (18). Of these six observables which involve θ, the decay rates $R_\pm$ involve also unknown decay amplitudes and, therefore, cannot be directly used for the determination of CPT violation in mixing.

In view of the previous difficulties [8–10] in achieving this last goal by using the decay rates $R_\pm$ for correlated decays of the C-even and C-odd $|M^0\bar{M}^0\rangle$ states, we have further investigated semileptonic decays in this context. We have shown in Sect. 7 (see also [8]) that the three complex parameters θ, λ_+ and $\bar{\lambda}_-$, where the latter two quantities parameterize violations of the $\Delta F = \Delta Q$ rule, may be separately determined either by comparing the time dependence of opposite-sign dilepton events from the state $|\psi_-\rangle$ with that from $|\psi_+\rangle$, or by considering both possible charges in the like-sign dilepton events from $|\psi_+\rangle$ alone. Note that, if one wishes to determine θ alone, it is sufficient to consider the time dependence of only $R_+(\ell^+, t_\ell; \ell^-, t_r)$ [8]. The disentanglement of the above-mentioned three parameters is – in principle – possible also by using semileptonic decays of single mesons M^0 and $\bar{M}^0$ (see [9] and Sect. 4); however, that requires initial-state tagging. As shown in Sect. 4, by considering the best presently available data [21], it is useful to have an alternative procedure which does not require flavor tagging. Our proposal for considering dilepton events from the decays of $|\psi_-\rangle$ *and* $|\psi_+\rangle$ may provide such an alternative.

Though some of the experiments proposed in this paper require difficult steps like flavor tagging and a study of the decay of the C-even $M^0\bar{M}^0$ state $|\psi_+\rangle$ [27], the importance of testing the fundamental property of CPT invariance may make the effort worthwhile.

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